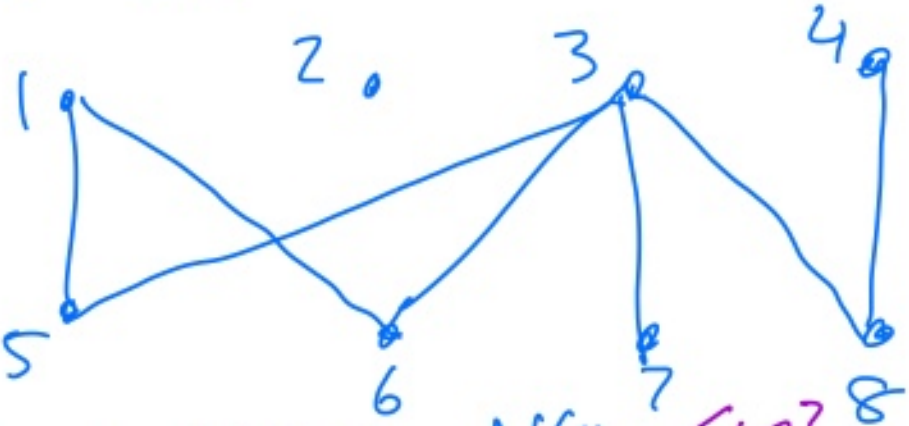
My graph (V, E)



What is $\deg(6)$? 2 $\quad N(6) = \{1, 3\}$

$\deg(7)$? 1 $\quad N(7) = \{3\}$

$\deg(8)$? 2 $\quad N(8) = \{3, 4\}$

$\deg(2)$? 0 $\quad N(2) = \emptyset$

If v is a vertex, $N(v)$ = "neighborhood of v"

$= \{w \in V : \text{there is a edge from } v \text{ to } w\}$

We can also define the neighborhood of a set S of vertices $(S \subseteq V)$:

$$N(S) = \bigcup_{v \in S} N(v)$$

$$N(\{1,2,3\}) = \{5,6\} \cup \emptyset \cup \{5,6,7,8\}$$

$$= \{5,6,7,8\}.$$

For a simple graph (no loops, no multiple edges),

$$|N(v)| = \deg(v)$$

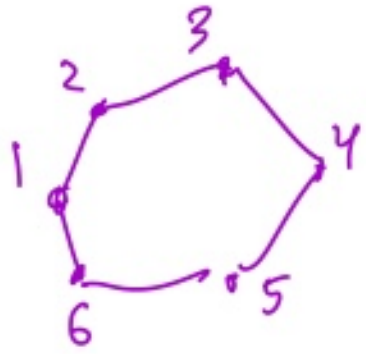
Special Kinds of Graphs:

① Complete graph K_n

n vertices

E = set of all possible sets of 2 vertices.

② Cyclic graph



C_6

$C_n = (V, E)$, $V = \{1, 2, \dots, n\}$

$E = \{\{1,2\}, \{2,3\}, \dots, \{n-1, n\}, \{n, 1\}\}$

③ Bipartite graph — a graph of the form (V, E), where $V = V_1 \amalg V_2$,

$\amalg$ ~ same as $\cup$, used when sets are disjoint

V_1, V_2: two disjoint sets

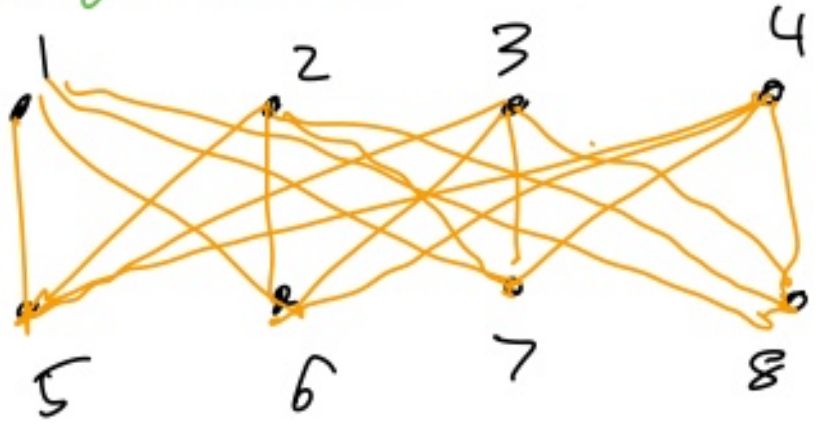
disjoint: $V_1 \cap V_2 = \emptyset$

and where all of the edges in E are of the form $\{a, b\}$ where $a \in V_1$, $b \in V_2$.

Complete Bipartite graph -

Bipartite graph that is simple and all possible edges from V_1 to V_2 are included.

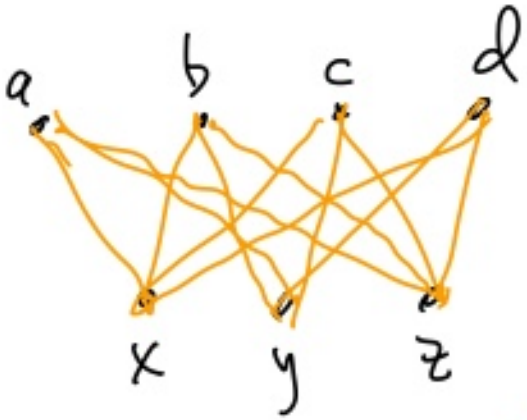
eg.



$K_{4,4}$

$|V_1|$ $|V_2|$

$K_{4,3}$



Question: In $K_{N,M}$, what is $|V|$? $N+M$

what is $|E|$? NM

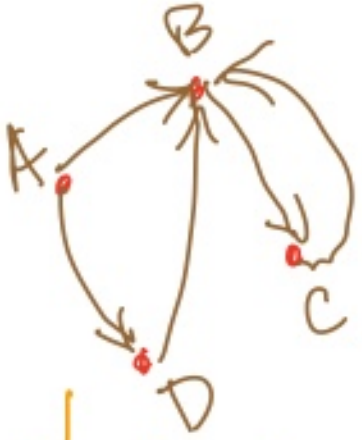
Complete bipartite graph with $V = V_1 \amalg V_2$, $|V_1| = N$, $|V_2| = M$.

Handshake theorem: If $G = (V, E)$ is an undirected, simple graph, then

$$\sum_{v \in V} \deg(v) = 2|E|$$

Note: This is still true if the graph has multiple edges but no loops.

For a directed, simple graph,



$$\sum_{v \in V} \deg^+(v) = |E|$$

$$\sum_{v \in V} \deg^-(v) = |E|$$

$$\sum_i \deg^+(v) = \overset{A}{2} + \overset{B}{1} + \overset{C}{1} + \overset{D}{1} = 5$$

$$\sum \deg^-(v) = 0 + 3 + 1 + 1 = 5$$

We say two vertices are <u>adjacent</u> if they are connected by an edge. (undirected graph).

In a directed graph, if there an edge from vertex a to vertex b, then we say b is adjacent to a.

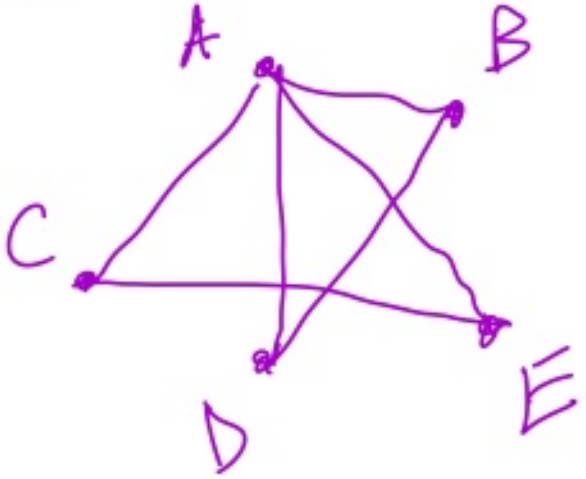
Adjacency Matrix will give us the information of which vertex goes to which other ones.

$$\begin{pmatrix} - & - & - \\ - & - & - \\ - & - & - \\ - & - & - \end{pmatrix}$$

Quiz #1

(1) What is your name?

(2) Let G be the graph below:



(a) Find the degree of each vertex.

(b) Is this a directed graph?

(3) Draw K_4, complete graph with 4 vertices.